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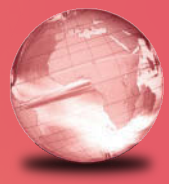

Calculus
& Its Applications

FOURTEENTH EDITION

Goldstein • Lay • Schneider • Asmar

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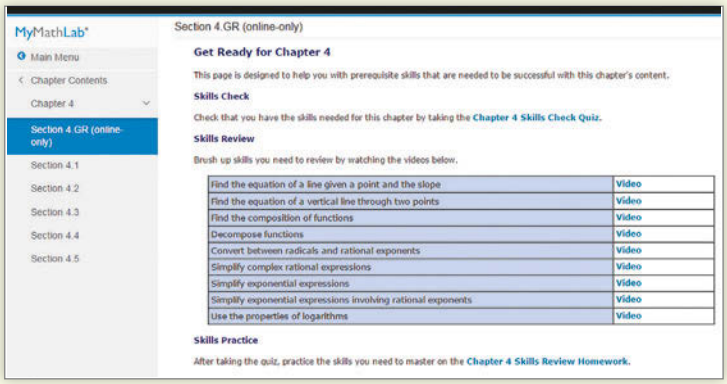

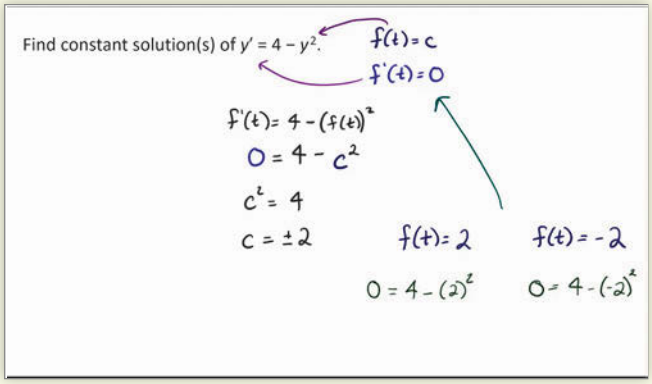

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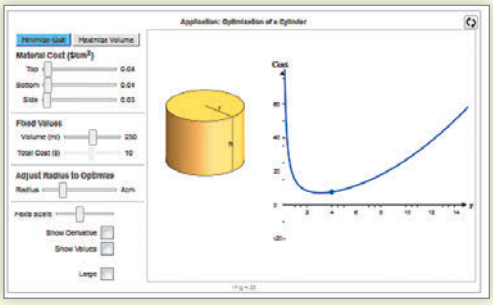

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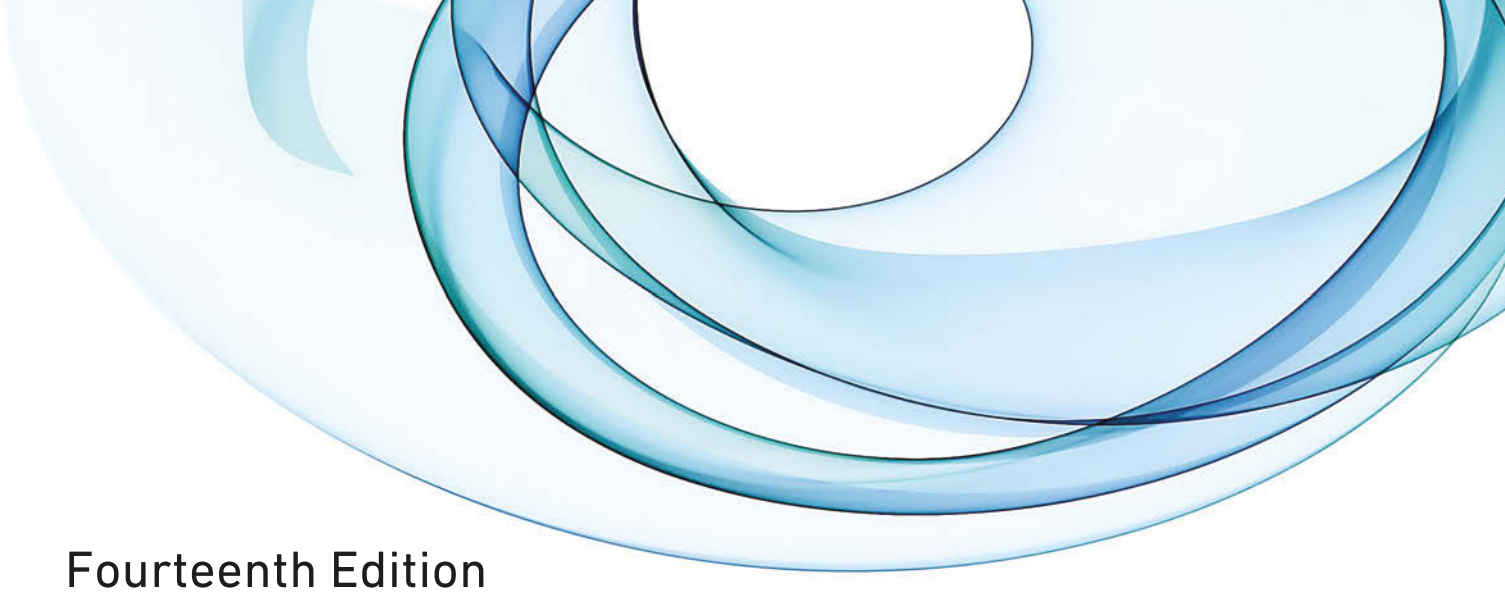

Fourteenth Edition
Global Edition

Calculus

& ITS APPLICATIONS

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Preface

This fourteenth edition of *Calculus & Its Applications* is written for either a one- or two-semester applied calculus course for students majoring in business or the social and biological sciences. Our goals for this revision reflect the original goals for the text, which include: to begin calculus as soon as possible; to present calculus in an intuitive yet intellectually satisfying way; and to integrate the many applications of calculus to business, life sciences, and social sciences. A primary goal of this revision was to make the calculus even more accessible and intuitive while maintaining the overall level and approach of previous editions.

Content

The distinctive order of topics has proven over the years to be very successful. A strength of our approach is to build intuition with key concepts of calculus before the analytical material. For instance, the derivative is explained geometrically before the analytical material on limits is presented. And the definite integral is introduced intuitively via the notion of net change (before the material on Riemann sums).

The significant applications introduced early in the course serve to motivate students and make the mathematics more accessible. To allow you to reach the applications in Chapter 2 quickly, we present only the differentiation rules and curve sketching needed.

Several texts in applied calculus over the past decade or so have shifted coverage of exponential and logarithmic functions to earlier in the text. We have maintained our later treatment because (a) it allows for undistracted focus on these important functions and their applications, and (b) it provides a venue to review important concepts (such as the Chain Rule) in a new context. Chapter 5, which covers applications of exponential and logarithmic functions, stands apart from all other texts for this course with its straightforward and intuitive use of differential equations to model a variety of phenomena. A key focus of this revision was to bolster the strengths of our approach in Chapters 4 and 5, making them even more understandable.

Because most courses do not afford enough time to cover all of the topics in this text and because schools have different goals for the course(s), we have been strategic with the placement and organization of topics. To this end, the level of theoretical material may be adjusted to meet the needs of the students. For example, Section 1.4 "Limits and the Derivative" may be omitted entirely if the instructor does not wish to present the notion of limit beyond the required material that is contained in Section 1.3. In addition, sections considered optional are starred in the table of contents.

Prerequisites

Because students often enter this course with a variety of prerequisite skills, Chapter 0 is available to either cover in its entirety or as a source of remediation, depending on the pace of the course. In addition to being covered in Chapter 0, some important topics, such as the laws of exponents, are reviewed again when they are used in a later chapter.

The Prerequisite Skills Diagnostic Test prior to Chapter 0 may help you in evaluating student gaps in prerequisite skills. The answers to the diagnostic test are provided in the student edition along with references to where students can go for remediation. Diagnostic tests and remediation are also provided in Pearson MyLab® Mathematics—with the added benefits of auto-grading and personalized remediation for any skill-gaps that are identified.

New to This Edition

We are grateful for the many helpful suggestions made by reviewers and users of the text. We incorporated many of these into this new edition. We also analyzed aggregated student usage and performance data from Pearson MyLab® Mathematics for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students. Additionally, we made the following improvements in this edition:

- Revised Examples to more closely align with the exercise sets.
- Added blue "help text" next to steps within worked-out examples to help students understand key algebraic or numerical transitions.
- In cases where properties or theorems that were formerly numbered (e.g., Property 4) have a commonly used name (e.g., Power of a Quotient Rule), we used the name rather than the number. This allows for more intuitive explanations within examples and is better aligned to how concepts are explained in class.
- Updated graphing calculator screens to TI-84 Plus CE (color) format.
- Added *For Review* features in the side margin to either remind students of a concept that is needed at that point in the book or direct the student back to the section in which it was covered earlier in the book.
- Revised all 3-dimensional figures in the text using the latest software. The difference in most cases is stunning, as can be seen in the sample figure here. We took full advantage of these updates to make the figures more effective pedagogically.

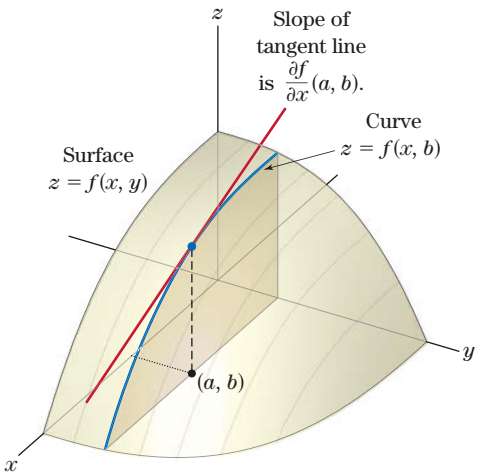


Figure 1 $\frac{\partial f}{\partial x}$ gives the slope of a curve formed by holding y constant at b.

- Updated data in examples and exercises. Many modern and student-centered applications have been added to help students see the relevance of the content.
- Added 225 new exercises and 30 new examples. The book now contains 4200 exercises and 520 worked-out examples.
- Updated examples and exercises to better reflect changes in culture and student population. And acknowledging that today's student population is more international than ever before, we changed the context of some problems to make them more understandable to students not born in the U.S.

New to Pearson MyLab® Mathematics

Many improvements have been made to the overall functionality of Pearson MyLab® Mathematics (MyLab) since the previous edition. However, beyond that, we have also increased and improved the content specific to this text.

- Instructors now have more exercises than ever to choose from in assigning homework. There are approximately 3300 assignable exercises in MyLab.
- The Conceptual Question Library provides additional conceptual questions for your use.
- The MyLab now contains a downloadable Instructor Answers document—*with all answers in one place*. (This augments the downloadable Instructor Solutions Manual, which contains all *solutions*.)
- Interactive Figures are now in HTML format (no plug-in required) and are supported by assignable exercises and tutorial videos.
- An Integrated Review version of the MyLab course contains pre-made quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified.
- New Setup & Solve exercises require students to show how they set up a problem as well as the solution, better mirroring what is required of students on tests.
- 97 new example-based videos were added.
- MathTalk videos highlight applications of the content of the course to business. The videos are supported by assignable exercises.
- Study skills modules help students with the life skills that can make the difference between passing and failing.
- Graphing Calculator and Excel Spreadsheet Manuals, specific to this course, are now downloadable from MyLab.

Content Changes

Chapter 0

- Added an example and exercises in 0.1 to illustrate the concept of piecewise-defined functions.
- Rewrote and simplified the introduction to 0.5 Exponents and Power Functions to make it more intuitive and easier to reference. Additionally, we added several examples to illustrate the rules of exponents.
- Added Examples 8 and 9 in 0.5 to illustrate the role of multiple factors in compound interest and investment accounts.
- Added Example 7 in 0.6 to illustrate various concepts from economics.
- Added four new exercises (45–48) in 0.6 to illustrate variations on the standard topic of compound interest.

Chapter 1

- Removed some of the proofs related to review material to simplify the presentation in 1.1.
- Added four new exercises (5–8) in 1.2 to illustrate the geometric meaning of the slope of a graph as the slope of the tangent line. Additionally, we modified two other exercises requiring reading and interpreting slopes of graphs.

- Simplified the discussion of limits in Examples 2 and 4 in 1.4.
- Included a discussion and a new Example 4 in 1.8 to illustrate the concepts of displacement and velocity.

Chapters 2 and 3

- Modified the Technology Exercises in 2.1 to make them more straightforward for students to answer.
- Improved and simplified the solutions within Example 4 in 2.4.
- Removed Example 4 in 2.6 which required more symbolic manipulation and use of constants than students would encounter in the exercises.
- Rewrote five examples in the Summary section of Chapter 2.
- Added ten new exercises in Chapter 3.

Chapter 4

- Revised Example 2 in 4.2 to better prepare students for the variety of exercises within the homework.
- Moved the material on the properties and graphs of exponential functions from 4.3 to 4.2.
- Replaced Examples 1, 2, and 3 from 4.3 with new examples that better build on the properties of derivatives introduced earlier. Example 3 introduces a new concept of combined returns to illustrate applications of linear combinations of exponential functions.
- Moved the material on differential equations in 4.3 to Chapter 5.
- Introduced forty new exercises in 4.3, including one on investment portfolios.
- Rewrote the introduction of 4.4 to better display and present the properties of logarithms.
- Changed Example 1 in 4.4 to better match the types of exercises in the homework.
- Used investment portfolios to illustrate applications involving solutions of equations with logarithms in Example 4 in 4.4.
- Rewrote the introduction of 4.5 to better display properties of the natural logarithm and its derivative.
- Added a new example on differentiation of the natural logarithm in 4.5.
- Modified and added over twenty exercises in 4.5.
- Rewrote the introduction of 4.6 to better present further properties of the logarithm.

Chapter 5

- Rewrote 5.1 to better show the applications of the derivative as a rate of change in setting up and solving differential equations. Changes to 5.1 also include:
  - Stated the solutions of the differential equation $y' = ky$ in a theorem.
  - Added an example on solving differential equations.
  - Stated the solutions of the initial value problem $y' = ky$, $y(0) = P_0$ in a theorem.
  - Illustrated the solutions of initial value problems with examples and a figure that a student can easily relate to.
  - Simplified the discussion on exponential decay and carbon dating.
  - Added twenty-three new exercises on differential equations and their applications.
- Rewrote and simplified the introduction to 5.2. Also in 5.2:
  - Introduced continuous compounding as a limit of the ordinary compounding from Chapter 0 and as a solution of a differential equation of the type that was discussed in 5.1.
  - Added narrative that compares continuous to ordinary compounding.
  - Added discussion, an example, and exercises on negative interest rates.
- Rewrote the introduction to 5.3 to better explain the concept of relative rate of change and how it relates to common pricing applications.
- Added an example in 5.3 on logarithmic derivatives.
- Included a summary of solutions of differential equations and their properties in the Chapter Summary.

Chapter 6

- Simplified Example 1 in 6.1 by adding more details to the solution.
- Rewrote the introduction to 6.2 and simplified the presentation by relating the new concept of definite integral to a common problem involving velocity and position.
- Simplified the numerical computations in Examples 3 and 5 in 6.2.
- Added nine new exercises to 6.2.
- Added a new example on online transportation in 6.4, as an application of area between two curves.

Chapter 7

- Revised all 3-dimensional figures in the chapter using the latest software.
- Added more help to Example 1 in 7.2 and added a 3-dimensional graph.
- In Example 4 in 7.3, added a new application of the second derivative test in two dimensions.
- Added twenty new exercises to 7.2, ten new exercises to 7.4, and two new exercises to 7.5.

Chapters 8–12

- Removed the application on population genetics from 10.6 (as this is now covered in Chapter 5).
- Revised exercises and examples to update real-world data.

Trusted Features

Though this edition has been improved in a variety of ways to reflect changing student needs, we have maintained the popular overall approach that has helped students be successful over the years.

Relevant and Varied Applications

We provide realistic applications that illustrate the uses of calculus in other disciplines and everyday life. The variety of applications is evident in the Index of Applications at the end of the text. Wherever possible, we attempt to use applications to motivate the mathematics. For example, our approach to the derivative in 1.3 is motivated by the slope formula and applications in 1.2, and applications of the net change functions in 6.2 motivate our approach to the definite integral in 6.2 and 6.3.

Plentiful Examples

The fourteenth edition includes 520 worked examples. Furthermore, we include computational details to enhance comprehension by students whose basic skills are weak. Knowing that students often refer back to examples for help, we built in fidelity between exercises and examples. In addition, students are given Now Try exercise references immediately following most examples to encourage them to check their understanding of the given example.

Exercises to Meet All Student Needs

The 4200 exercises are the most important part of the text, in our opinion. The exercises at the ends of the sections are typically arranged in the order in which the text proceeds, so that homework assignments may be made easily after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Exercises have odd-even pairing, when appropriate. Chapter Review Exercises are designed to prepare students for end-of-chapter tests. Answers to the odd-numbered exercises, and all Chapter Review Exercises, are included at the back of the book.

Check Your Understanding Problems

The Check Your Understanding problems are a popular and useful feature of the book. They are carefully selected exercises located at the end of each section, just before the exercise set. Complete solutions follow the exercise set. These problems prepare students for the exercise sets beyond just covering simple examples. They give students a chance to think about the skills they are about to apply and reflect on what they've learned.

Use of Technology

As in previous editions, the use of graphing calculators is not required for the study of this text; however, for instructors who wish to incorporate them we have included instructions for their use in the Integrating Technology features within sections. In our discussions of graphing calculators, we specifically refer to the TI-84 Plus models. New to this edition, screen shots display the new color versions of the TI-84. Most exercise sets include problems at the end labeled Technology Exercises.

End-of-Chapter Study Aids

Near the end of each chapter is a set of problems entitled Fundamental Concept Check Exercises that help students recall key ideas of the chapter and focus on the relevance of these concepts as well as prepare for exams. Each chapter also contains a two-column grid giving a section-by-section summary of key terms and concepts with examples. Finally, each chapter has Chapter Review Exercises that provide more practice and preparation for chapter-level exams.

Technology and Supplements

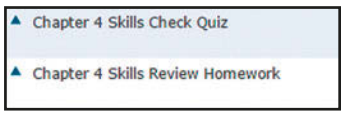
Pearson MyLab® Mathematics Online Course (access code required)

Built around Pearson's best-selling content, Pearson MyLab® Mathematics is an online homework, tutorial, and assessment program designed to work with this text to engage students and improve results. Pearson MyLab® Mathematics can be successfully implemented in any classroom environment—lab-based, hybrid, fully online, or traditional. **By addressing instructor and student needs, Pearson MyLab® Mathematics improves student learning.**

Used by more than 37 million students worldwide, Pearson MyLab® Mathematics delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success. Visit www.mymathlab.com/results to learn more.

Preparedness

One of the biggest challenges in applied calculus courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. Pearson offers a variety of content and course options to support students with just-in-time remediation and key-concept review.

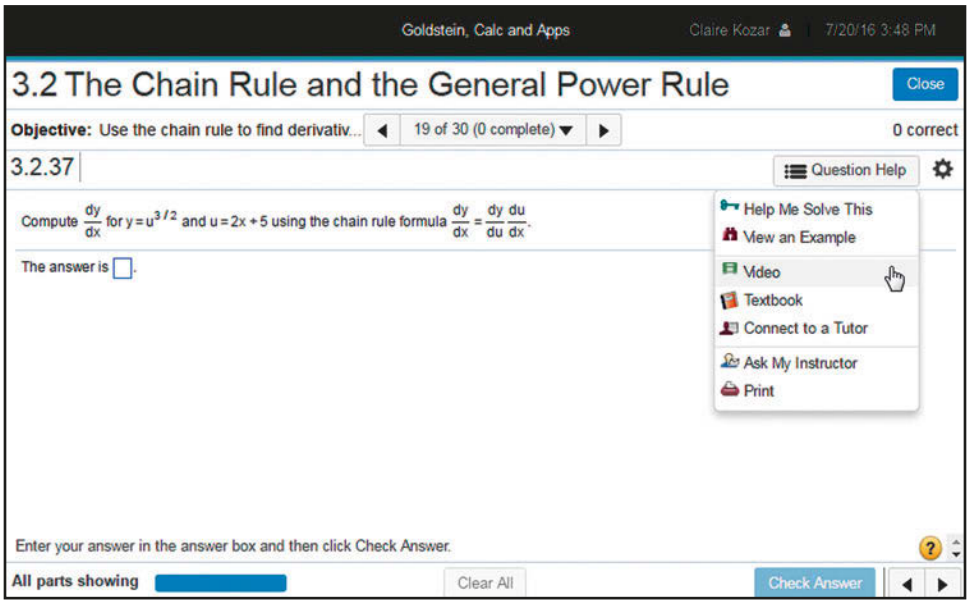


- **Integrated Review Courses** can be used for just-in-time prerequisite review. These courses provide additional prerequisite content on review topics, along with pre-made, assignable skill-check quizzes, personalized homework assignments, and videos integrated throughout the course content.

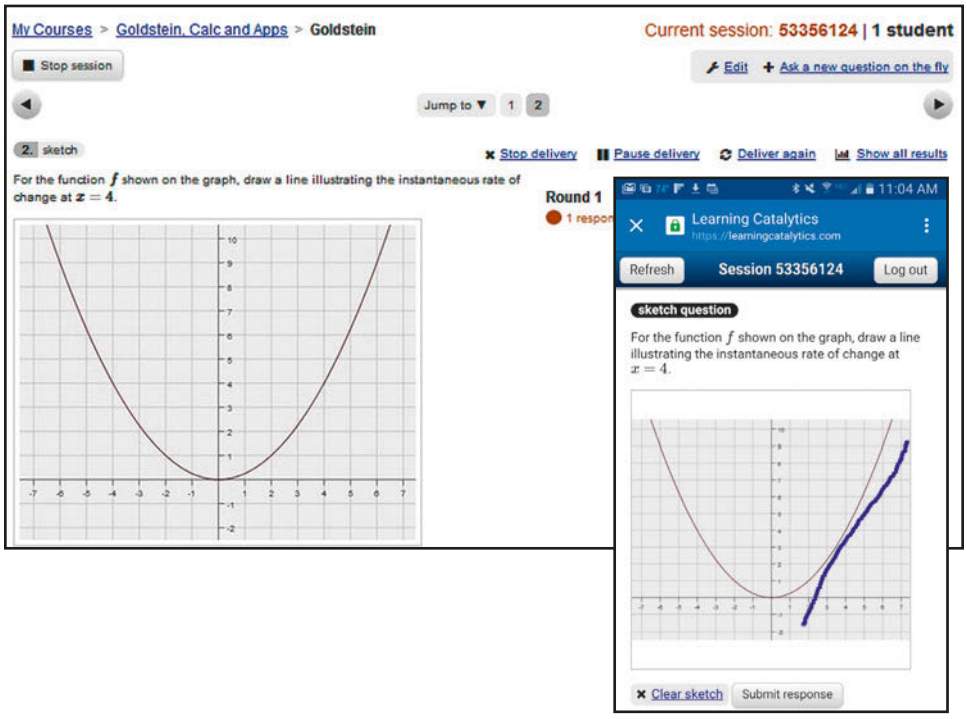
Motivation

Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of mathematics. Pearson MyLab® Mathematics online homework offers students immediate feedback and tutorial assistance that motivates them to do more, which means they retain more knowledge and improve their test scores.

- **Exercises with immediate feedback**—over 3300 assignable exercises—are based on the textbook exercises, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Pearson MyLab® Mathematics provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and an eText.

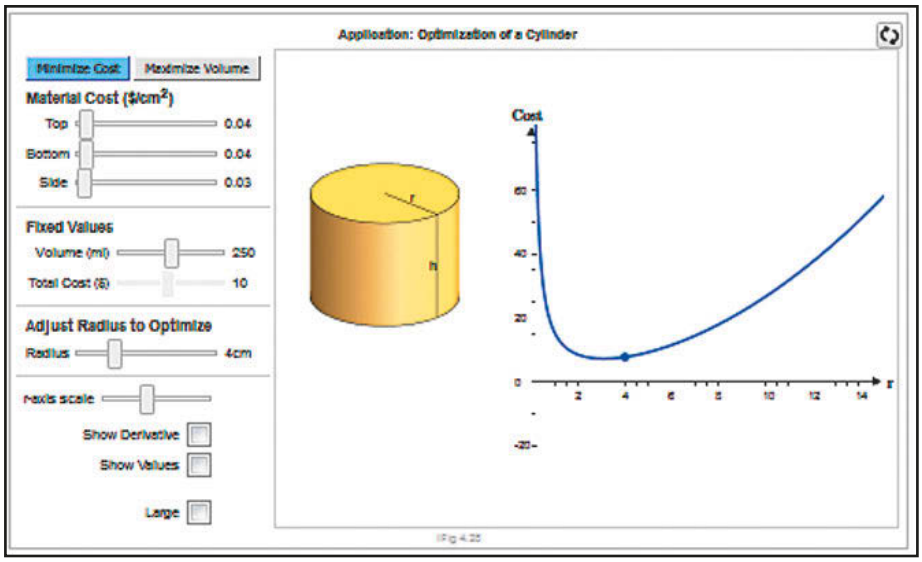


- **Setup and Solve Exercises** ask students to first describe how they will set up and approach the problem. This reinforces students' conceptual understanding of the process they are applying and promotes long-term retention of the skill.
- **Conceptual Question Library** exercises require students to apply their conceptual understanding to solve problems—mostly those involving applications.
- **MathTalk videos** connect the math to the real world (particularly business). The videos include assignable exercises to gauge students' understanding of video content.
- **Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics.



Learning and Teaching Tools

- **Interactive Figures** illustrate key concepts and allow manipulation for use as teaching and learning tools. Includes assignable exercises that require use of figures and instructional videos that explain the concept behind each figure.



- **Instructional videos**—277 example-based videos—are available as learning aids within exercises and for self-study. The Guide to Video-Based Assignments makes it easy to assign videos for homework by showing which Pearson MyLab® Mathematics exercises correspond to each video.
- **Complete eText** is available to students through their Pearson MyLab® Mathematics courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook.
- **Skills for Success Modules** helps students with the life skills that can make the difference between passing and failing. Topics include "Time Management" and "Stress Management."
- **Excel Spreadsheet Manual,** specifically written for this course.
- **Graphing Calculator Manual,** specifically written for this course.
- **PowerPoint Presentations** are available for download for each section of the book.
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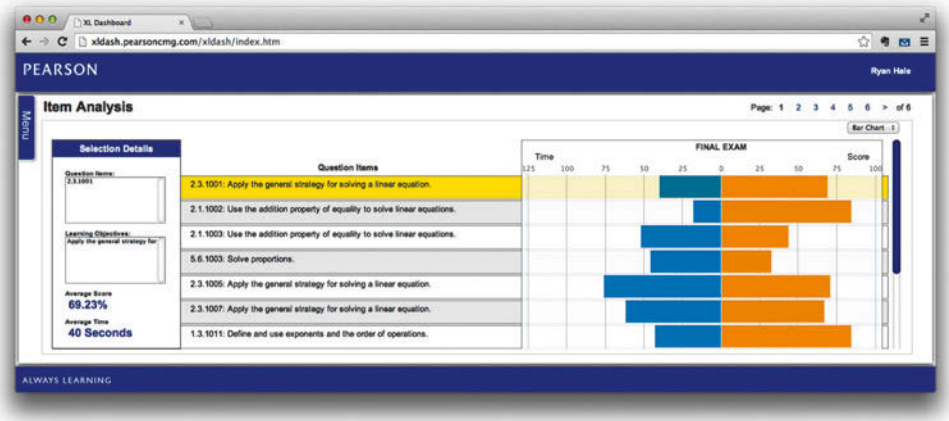

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Contains classroom presentation slides for this textbook featuring lecture content, worked-out examples, and key graphics from the text. Available to qualified instructors within Pearson MyLab® Mathematics or through the Pearson Instructor Resource Center at www.pearsonglobaleditions.com/goldstein.

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Prerequisite Skills Diagnostic Test

To the Student and the Instructor Are you ready for calculus? This prerequisite skills diagnostic test evaluates basic mathematical skills that are required to begin the course. It is not intended to replace a placement test that your institution may already have. Each set of questions refers to a section in Chapter 0 of the text. If you miss several questions from one part, you may want to study the corresponding section from Chapter 0.

Calculate the given quantities using the laws of exponents. **(Section 0.5)**

1. $\dfrac{7^{3/2}}{49}\sqrt{7}$

2. $(3^{1/3}3^{1/2})^2\sqrt[3]{3}$

3. $\dfrac{\sqrt{5}}{\sqrt{15}\sqrt{3}}$

4. $2^{1/3}2^{1/2}2^{1/6}$

Simplify the given expressions. Your answer should not involve parentheses or negative exponents. **(Section 0.5)**

5. $\dfrac{x^2}{x^{-4}}$

6. $\dfrac{x^2(x^{-4}+1)}{x^{-2}}$

7. $\left(\dfrac{x}{x^2y^2}\right)^3 y^8$

8. $\left(\dfrac{1}{xy}\right)^{-2}\left(\dfrac{x}{y}\right)^2$

Given $f(x)=\dfrac{x}{x+1}$ and $g(x)=x+1$, express the following as rational functions. **(Section 0.3)**

9. $f(x)+g(x)$

10. $f(x)g(x)$

11. $\dfrac{g(x)}{f(x)}$

12. $f(x)-\dfrac{g(x)}{x+1}$

Given $f(t)=t^2$ and $g(t)=\dfrac{t}{t+1}$, calculate the following functions. Simplify your answer as much as possible. **(Section 0.3)**

13. $f(g(t))$

14. $g(f(t))$

15. $f(f(g(t)))$

16. $f(g(t+1))$

For the given $f(x)$, find $\dfrac{f(x+h)-f(x)}{h}$ and simplify your answer as much as possible. **(Section 0.3)**

17. $f(x)=x^2+2x$

18. $f(x)=\dfrac{1}{x}$

19. $f(x)=\sqrt{x}$ (*Hint:* Rationalize the numerator.)

20. $f(x)=x^3-1$

Graph the following functions. Determine clearly the intercepts of the graphs. **(Section 0.2)**

21. $f(x)=2x-1$

22. $f(x)=-x$

23. $f(x)=-\dfrac{x-1}{2}$

24. $f(x)=3$

Find the points of intersection (if any) of the pairs of curves. **(Section 0.4)**

25. $y=3x+1, y=-x-2$

26. $y=\dfrac{x}{2}, y=3$

27. $y=4x-7, y=0$

28. $y=2x+3, y=2x-2$

Factor the given polynomials. **(Section 0.4)**

29. $x^2 + 5x - 14$

30. $x^2 + 5x + 4$

31. $x^3 + x^2 - 2x$

32. $x^3 - 2x^2 - 3x$

Solve the given equations by factoring first. **(Section 0.4)**

33. $x^2 - 144 = 0$

34. $x^2 + 4x + 4 = 0$

35. $x^3 + 8x^2 + 15x = 0$

36. $6x^3 + 11x^2 + 3x = 0$

Solve using the quadratic formula. **(Section 0.4)**

37. $2x^2 + 3x - 1 = 0$

38. $x^2 + x - 1 = 0$

39. $-3x^2 + 2x - 4 = 0$

40. $x^2 + 4x - 4 = 0$

Solve the given equations. **(Section 0.4)**

41. $x^2 - 3x = 4x - 10$

42. $4x^2 + 2x = -2x + 3$

43. $\dfrac{1}{x+1} = x + 1$

44. $\dfrac{x^3}{x^2 + 2x - 1} = x - 1$

Introduction

Often, it is possible to give a succinct and revealing description of a situation by drawing a graph. For example, Fig. 1 describes the amount of money in a bank account drawing 5% interest, compounded daily. The graph shows that, as time passes, the amount of money in the account grows. Figure 2 depicts the weekly sales of a breakfast cereal at various times after advertising has ceased. The graph shows that the longer the time since the last advertisement, the fewer the sales. Figure 3 shows the size of a bacteria culture at various times. The culture grows larger as time passes. But there is a maximum size that the culture cannot exceed. This maximum size reflects the restrictions imposed by food supply, space, and similar factors. The graph in Fig. 4 describes the decay of the radioactive isotope iodine 131. As time passes, less and less of the original radioactive iodine remains.

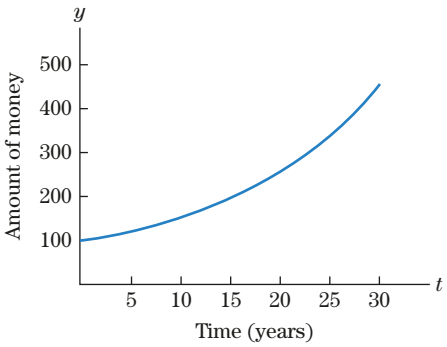


Figure 1 Growth of money in a savings account.

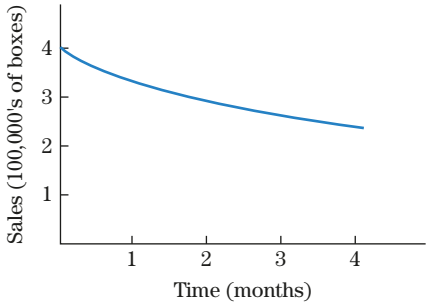


Figure 2 Decrease in sales of breakfast cereal.

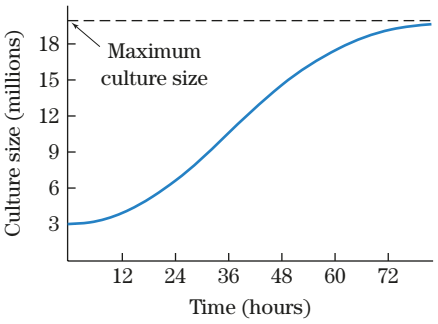


Figure 3 Growth of a bacteria culture.

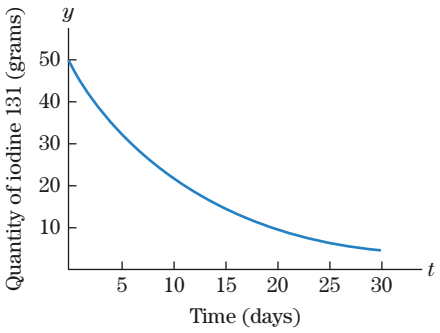


Figure 4 Decay of radioactive iodine.

Each graph in Figs. 1 to 4 describes a change that is taking place. The amount of money in the bank is changing, as are the sales of cereal, the size of the bacteria culture, and the amount of iodine. Calculus provides mathematical tools to study each change in a quantitative way.

CHAPTER

0

Functions

0.1 Functions and Their Graphs
0.2 Some Important Functions
0.3 The Algebra of Functions
0.4 Zeros of Functions—The Quadratic Formula and Factoring
0.5 Exponents and Power Functions
0.6 Functions and Graphs in Applications

Each graph in Figures 1 through 4 of the introduction depicts a relationship between two quantities. For example, Fig. 4 illustrates the relationship between the quantity of iodine (measured in grams) and time (measured in days). The basic quantitative tool for describing such relationships is a **function**. In this preliminary chapter, we develop the concept of a function and review important algebraic operations on functions used later in the text.

0.1 Functions and Their Graphs

Real Numbers

Most applications of mathematics use real numbers. For purposes of such applications (and the discussions in this text), it suffices to think of a real number as a decimal. A **rational number** is one that may be written as a fraction, and consequently as a finite or infinitely repeating decimal, such as

$$-\frac{5}{2} = -2.5, \qquad 1, \qquad \frac{13}{3} = 4.333\ldots \qquad \text{Rational numbers.}$$

An **irrational number** cannot be represented as a fraction, but instead has an infinite decimal representation whose digits form no repeating pattern, such as

$$-\sqrt{2} = -1.414213\ldots, \qquad \pi = 3.14159\ldots \qquad \text{Irrational numbers.}$$

The real numbers are described geometrically by a **number line**, as in Fig. 1. Each number corresponds to one point on the line, and each point determines one real number.

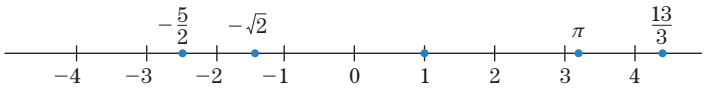


Figure 1 The real number line.

We use four types of inequalities to compare real numbers.

$x < y$	x is less than y
$x \le y$	x is less than or equal to y
$x > y$	x is greater than y
$x \ge y$	x is greater than or equal to y

The double inequality $a < b < c$ is shorthand for the pair of inequalities $a < b$ and $b < c$. Similar meanings are assigned to other double inequalities, such as $a \le b < c$. The three numbers in a double inequality, such as $1 < 3 < 4$ or $4 > 3 > 1$, should have the same relative positions on the number line as in the inequality. Thus $3 < 4 > 1$ is never written because the numbers are "out of order."

Geometrically, the inequality $x \le b$ means that either x equals b or x lies to the left of b on the number line. The set of real numbers x that satisfies the double inequality $a \le x \le b$ corresponds to the line segment between a and b, including the endpoints. This set is sometimes denoted by $[a, b]$ and is called the **closed interval** from a to b. If a and b are removed from the set, the set is written as (a, b) and is called the **open interval** from a to b. The notation for various line segments is listed in Table 1. Note that an interval endpoint that is included (e.g., both endpoints of $[a, b]$) is drawn as a solid circle, whereas an endpoint not included (e.g., the endpoint a in $(a, b]$) is drawn as an unfilled circle.

Table 1 Intervals on the Number Line

Inequality	Geometric Description	Interval Notation
$a \le x \le b$		$[a, b]$
$a < x < b$		(a, b)
$a \le x < b$		$[a, b)$
$a < x \le b$		$(a, b]$
$a \le x$		$[a, \infty)$
$a < x$		(a, ∞)
$x \le b$		$(-\infty, b]$
$x < b$		$(-\infty, b)$

The symbols ∞ ("infinity") and $-\infty$ ("minus infinity") do not represent actual real numbers. Rather, they indicate that the corresponding line segment extends infinitely far to the right or left. An inequality that describes such an infinite interval may be written in two ways. For instance, $a \le x$ is equivalent to $x \ge a$.

EXAMPLE 1 **Graphing Intervals** Describe each of the following intervals both graphically and in terms of inequalities.

(a) $(-1, 2)$ **(b)** $[-2, \pi]$ **(c)** $(2, \infty)$ **(d)** $(-\infty, \sqrt{2}]$

SOLUTION The line segments and the inequalities corresponding to the intervals are shown in Fig. 2(a) through (d).

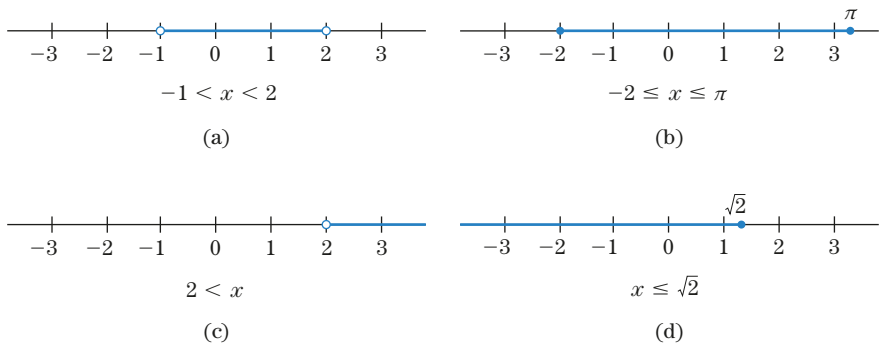


Figure 2 Intervals on the Number Line

»» *Now Try Exercise 7*

EXAMPLE 2 **Using Inequalities** Let the variable x describe the profit that a company anticipates to earn in the current fiscal year. The business plan calls for a profit of at least \$5 million. Describe this aspect of the business plan in the language of intervals.

SOLUTION The phrase "at least" means "greater than or equal to." The business plan requires that $x \geq 5$ (where the units are millions of dollars). This is equivalent to saying that x lies in the infinite interval $[5, \infty)$. «

Functions

A **function** of a variable x is a rule f that assigns each value of x to a unique number, $f(x)$, called the **value of the function at** x. (We read "$f(x)$" as "f of x".) The variable of a function is called the **independent variable**. The set of values that the independent variable is allowed to assume is called the **domain** of the function. The domain of a function may be explicitly specified as part of the definition of a function, or it may be understood from context, as you will see in Examples 5 and 7. The **range** of a function is the set of values that the function assumes.

The functions we shall meet in this book will usually be defined by algebraic formulas. For example, the domain of the function

$$f(x) = 3x - 1$$

consists of all real numbers x. This function is the rule that takes a number, multiplies it by 3, and then subtracts 1. If we specify a value of x—say, $x = 2$—then we find the value of the function at 2 by substituting 2 for x in the formula:

$$f(2) = 3(2) - 1 = 5.$$

EXAMPLE 3 **Evaluating a Function** Let f be the function with domain all real numbers x and defined by the formula

$$f(x) = 3x^3 - 4x^2 - 3x + 7.$$

Find $f(2)$ and $f(-2)$.

SOLUTION To find $f(2)$, we substitute 2 for every occurrence of x in the formula for $f(x)$:

$$\begin{aligned} f(2) &= 3(2)^3 - 4(2)^2 - 3(2) + 7 && \text{Substitute 2 for } x. \\ &= 3(8) - 4(4) - 3(2) + 7 && \text{Evaluate exponents.} \\ &= 24 - 16 - 6 + 7 && \text{Multiply.} \\ &= 9. && \text{Add and subtract.} \end{aligned}$$

To find $f(-2)$, we substitute (-2) for each occurrence of x in the formula for $f(x)$. The parentheses ensure that the -2 is substituted correctly. For instance, x^2 must be replaced by $(-2)^2$, not -2^2:

$$\begin{aligned} f(-2) &= 3(-2)^3 - 4(-2)^2 - 3(-2) + 7 && \text{Substitute } (-2) \text{ for } x. \\ &= 3(-8) - 4(4) - 3(-2) + 7 && \text{Evaluate exponents.} \\ &= -24 - 16 + 6 + 7 && \text{Multiply.} \\ &= -27. && \text{Add and subtract.} \end{aligned}$$

»» *Now Try Exercise 13*

EXAMPLE 4 **Temperature Scales** If x represents the temperature of an object in degrees Celsius, then the temperature in degrees Fahrenheit is a function of x, given by $f(x) = \frac{9}{5}x + 32$.

(a) Water freezes at 0°C (C = Celsius) and boils at 100°C. What are the corresponding temperatures in degrees Fahrenheit (F = Fahrenheit)?

(b) Aluminum melts at 660°C. What is its melting point in degrees Fahrenheit?

SOLUTION

(a) $f(0) = \frac{9}{5}(0) + 32 = 32$. Water freezes at 32°F.

$f(100) = \frac{9}{5}(100) + 32 = 180 + 32 = 212$. Water boils at 212°F.

(b) $f(660) = \frac{9}{5}(660) + 32 = 1188 + 32 = 1220$. Aluminum melts at 1220°F.

»» *Now Try Exercise 19*

In the preceding examples, the functions had domains consisting of all real numbers or an interval. For some functions, the domain may consist of several intervals, with a different formula defining the function on each interval. Here is an illustration of this phenomenon.

EXAMPLE 5 **A Function Defined by Multiple Formulas** A company that runs an online movie streaming website charges users \$3 per movie. If a user streams 10 or more movies in one month, the company adds a 10% overusage fee to the original cost. Let x denote the number of movies a user streams a month and let $f(x)$ be the amount of dollars the company charges the user as a function of x.

(a) Describe $f(x)$.

(b) Find $f(5)$ and $f(15)$.

SOLUTION

(a) The formula $f(x)$ depends on whether $0 \le x < 10$ or $10 \le x$. When $0 \le x < 10$, the charge is $3x$ dollars. When $10 \le x$, the charge is $3x$ plus 10% of $3x$ or $3x + 0.1(3x) = 3.3x$ dollars. The domain consists of the values of x in one of the two intervals $[0, 10)$ and $[10, \infty)$. In each of these intervals, the function is defined by a separate formula:

$$f(x) = \begin{cases} 3x & \text{for } 0 \le x < 10 \\ 3.3x & \text{for } 10 \le x. \end{cases}$$

(b) Since $x = 5$ satisfies $0 \le x < 10$, we use the first formula for $f(x)$: $f(5) = 3(5) = 15$. So it costs \$15 to stream 5 movies in one month. Since $x = 15$ satisfies $10 \le x$, we use the second formula for $f(x)$: $f(15) = 3.3(15) = 49.5$. So it costs \$49.50 to stream 15 movies in one month.

»» *Now Try Exercise 57*

In calculus, it is often necessary to substitute an algebraic expression for x and simplify the result, as illustrated in the following example.

EXAMPLE 6 **Evaluating a Function** If $f(x) = (4 - x)/(x^2 + 3)$, what is (a) $f(a)$? (b) $f(a + 1)$?

SOLUTION

(a) Here, a represents some number. To find $f(a)$, we substitute a for x wherever x appears in the formula defining $f(x)$:

$$f(a) = \frac{4 - a}{a^2 + 3}.$$

(b) To evaluate $f(a + 1)$, substitute $a + 1$ for each occurrence of x in the formula for $f(x)$:

$$f(a + 1) = \frac{4 - (a + 1)}{(a + 1)^2 + 3}.$$

We can simplify the expression for $f(a + 1)$ using the fact that $(a + 1)^2 = (a + 1)(a + 1) = a^2 + 2a + 1$:

Expand Add and Subtract

$$f(a + 1) = \frac{4 - (a + 1)}{(a + 1)^2 + 3} = \frac{4 - a - 1}{a^2 + 2a + 1 + 3} = \frac{3 - a}{a^2 + 2a + 4}.$$ «

More about the Domain of a Function When defining a function, it is necessary to specify the domain of the function, which is the set of acceptable values of the variable. In the preceding examples, we explicitly specified the domains of the functions considered. However, throughout the remainder of the text, we will usually mention functions without specifying domains. In such circumstances, we will understand the intended domain to consist of all numbers for which the defining formula(s) makes sense. For example, consider the function

$$f(x) = x^2 - x + 1.$$

The expression on the right may be evaluated for any value of x. So, in the absence of any explicit restrictions on x, the domain is understood to consist of all numbers. As a second example, consider the function

$$f(x) = \frac{1}{x}.$$

Here x may be any number except zero (division by zero is not permissible.) So the domain intended is the set of nonzero numbers. Similarly, when we write

$$f(x) = \sqrt{x},$$

we understand the domain of $f(x)$ to be the set of all nonnegative numbers, since the square root of a real number x is defined if and only if $x \geq 0$.

EXAMPLE 7 **Domains of Functions** Find the domains of the following functions:

(a) $f(x) = \sqrt{4 + x}$ **(b)** $g(x) = \dfrac{1}{\sqrt{1 + 2x}}$ **(c)** $h(x) = \sqrt{1 + x} - \sqrt{1 - x}$

SOLUTION

(a) Since we cannot take the square root of a negative number, we must have $4 + x \geq 0$, or equivalently, $x \geq -4$. So the domain of f is $[-4, \infty)$.

(b) Here, the domain consists of all x for which

$$\begin{aligned} 1 + 2x &> 0 \\ 2x &> -1 \qquad \text{Subtract 1 from both sides.} \\ x &> -\frac{1}{2} \qquad \text{Divide both sides by 2.} \end{aligned}$$

The domain is the open interval $\left(-\frac{1}{2}, \infty\right)$.

(c) In order to be able to evaluate both square roots that appear in the expression of $h(x)$, we must have

$$1 + x \geq 0 \quad \text{and} \quad 1 - x \geq 0.$$

The first inequality is equivalent to $x \geq -1$, and the second inequality to $x \leq 1$. Since x must satisfy both inequalities, it follows that the domain of h consists of the closed interval $[-1, 1]$. »» Now Try Exercise 23

Graphs of Functions Often it is helpful to describe a function f geometrically, using a rectangular xy-coordinate system. Given any x in the domain of f, we can plot the point $(x, f(x))$. This is the point in the xy-plane whose y-coordinate is the value of the function at x. The set of *all* such points $(x, f(x))$ usually forms a curve in the xy-plane and is called the **graph** of the function $f(x)$.

It is possible to approximate the graph of $f(x)$ by plotting the points $(x, f(x))$ for a representative set of values of x and joining them by a smooth curve. (See Fig. 3.) The more closely spaced the values of x, the closer the approximation.

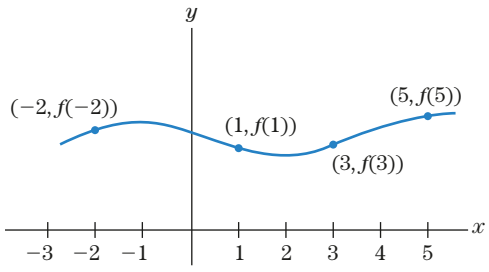


Figure 3

EXAMPLE 8 **Sketching a Graph by Plotting Points** Sketch the graph of the function $f(x) = x^3$.

SOLUTION The domain consists of all real numbers x. We choose some representative values of x and tabulate the corresponding values of $f(x)$. We then plot the points $(x, f(x))$ and draw a smooth curve through the points. (See Fig. 4.)

x	$f(x) = x^3$
0	0
1	1
2	8
3	27
−1	−1
−2	−8
−3	−27

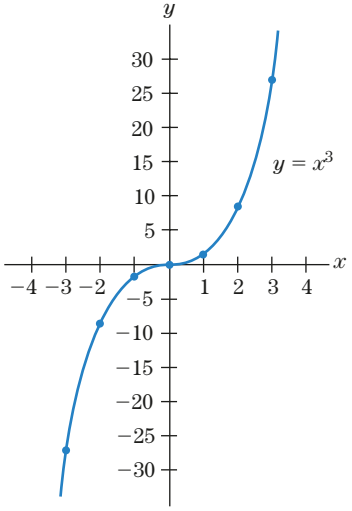


Figure 4 Graph of $f(x) = x^3$.

«

EXAMPLE 9 **A Graph with a Restricted Domain** Sketch the graph of the function $f(x) = 1/x$.

SOLUTION The domain of the function consists of all numbers except zero. The table in Fig. 5 lists some representative values of x and the corresponding values of $f(x)$. A function often has interesting behavior for x near a number not in the domain. So, when we chose representative values of x from the domain, we included some values close to zero. The points $(x, f(x))$ are plotted and the graph sketched in Fig. 5.

x	$f(x) = \frac{1}{x}$
$\frac{1}{4}$	4
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$

x	$f(x) = \frac{1}{x}$
$-\frac{1}{4}$	-4
$-\frac{1}{2}$	-2
-1	-1
-2	$-\frac{1}{2}$
-3	$-\frac{1}{3}$
-4	$-\frac{1}{4}$

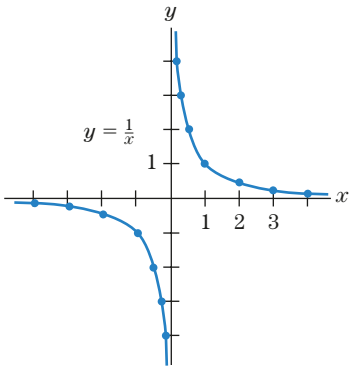


Figure 5 Graph of $f(x) = \frac{1}{x}$.

» Now Try Exercise 27

Now that graphing calculators and computer graphing programs are widely available, we seldom need to sketch graphs by hand-plotting a large number of points on graph paper. However, to use such a calculator or program effectively, we must know in advance which part of a curve to display. Critical features of a graph may be missed or misinterpreted if, for instance, the scale on the x- or y-axis is inappropriate.

An important use of calculus is to identify key features of a function that should appear in its graph. In many cases, only a few points need be plotted, and the general shape of the graph is easy to sketch by hand. For more complicated functions, a graphing program is helpful. Even then, calculus provides a way of checking that the graph on the computer screen has the correct shape. Algebraic calculations are usually part of the analysis. The appropriate algebraic skills are reviewed in this chapter.

Note also that analytic solutions to problems typically provide more precise information than graphing calculators and can provide insight into the behavior of the functions involved in the solution.

The connection between a function and its graph is explored in this section and in Section 0.6.

EXAMPLE 10 **Reading a Graph** Suppose that f is the function whose graph is given in Fig. 6. Notice that the point $(x, y) = (3, 2)$ is on the graph of f.

(a) What is the value of the function when $x = 3$?

(b) Find $f(-2)$.

(c) What is the domain of f? What is its range?

SOLUTION

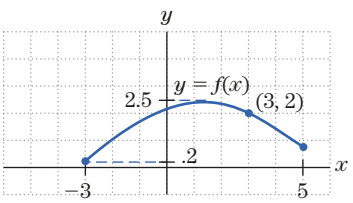


Figure 6

(a) Since (3, 2) is on the graph of f, the y-coordinate 2 must be the value of f at the x-coordinate 3. That is, $f(3) = 2$.

(b) To find $f(-2)$, we look at the y-coordinate of the point on the graph where $x = -2$. From Fig. 6, we see that $(-2, 1)$ is on the graph of f. Thus, $f(-2) = 1$.

(c) The points on the graph of $f(x)$ all have x-coordinates between -3 and 5 inclusive; and for each value of x between -3 and 5, there is a point $(x, f(x))$ on the graph. So the domain consists of those x for which $-3 \le x \le 5$. From Fig. 6, the function assumes all values between .2 and 2.5. Thus, the range of f is [.2, 2.5].

» Now Try Exercise 37

As we saw in Example 10, the graph of f can be used to picture the domain of f on the x-axis and its range on the y-axis. The general situation is illustrated in Fig. 7.

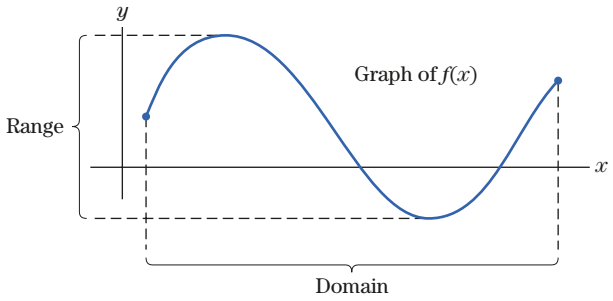


Figure 7 Domain and range.

To every x in the domain, a function assigns one and only one value of y, that is, the function value $f(x)$. This implies, among other things, that not every curve is the graph of a function. To see this, refer first to the curve in Fig. 6, which *is* the graph of a function. It has the following important property: For each x between -3 and 5 inclusive, there is a *unique* y such that (x, y) is on the curve. The variable y is called the **dependent variable**, since its value depends on the value of the independent variable x. Refer to the curve in Fig. 8. It cannot be the graph of a function because a function f must assign to each x in its domain a *unique* value $f(x)$. However, for the curve of Fig. 8, there corresponds to $x = 3$ (for example) more than one y-value: $y = 1$ and $y = 4$.

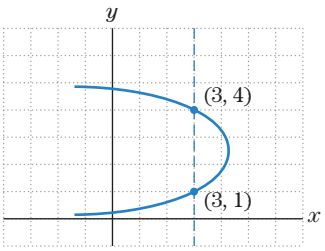


Figure 8 A curve that is *not* the graph of a function.

The essential difference between the curves in Figs. 6 and 8 leads us to the following test.

The Vertical Line Test A curve in the xy-plane is the graph of a function if and only if each vertical line cuts or touches the curve at no more than one point.

EXAMPLE 11 **Vertical Line Test** Which of the curves in Fig. 9 are graphs of functions?

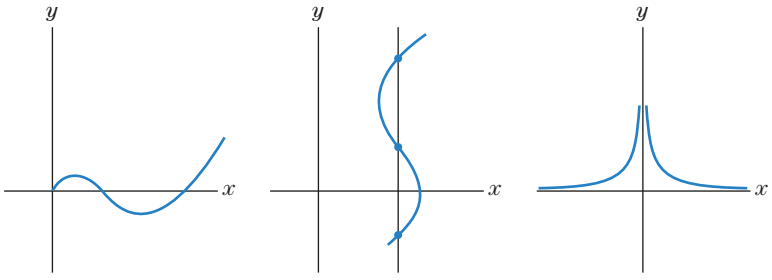


Figure 9

SOLUTION The curve in (a) is the graph of a function. It appears that vertical lines to the left of the y-axis do not touch the curve at all. This simply means that the function represented in (a) is defined only for $x \geq 0$. The curve in (b) is *not* the graph of a function because some vertical lines cut the curve in three places. The curve in (c) is the graph of a function whose domain is all nonzero x. (There is no point on the curve in (c) whose x-coordinate is 0.)

»» Now Try Exercise 29

There is another notation for functions that we will find useful. Suppose that $f(x)$ is a function. When $f(x)$ is graphed on an xy-coordinate system, the values of $f(x)$ give the y-coordinates of points of the graph. For this reason, the function is often abbreviated by the letter y, and we find it convenient to speak of "the function $y = f(x)$." For

example, the function $y = 2x^2 + 1$ refers to the function $f(x)$ for which $f(x) = 2x^2 + 1$. The graph of a function $f(x)$ is often called the **graph of the equation** $y = f(x)$.

INCORPORATING TECHNOLOGY

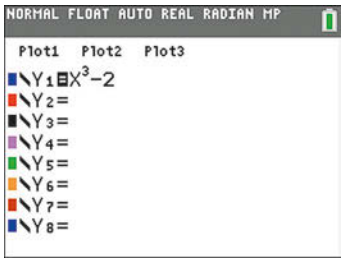
Graphing Functions The use of graphing calculators is not required for the study of this text; however, graphing calculators are very useful tools that can be used to simplify computations, draw graphs, and sometimes enhance understanding of the fundamental topics of calculus. Helpful information about the use of calculators will appear at the end of some sections in subsections titled "Incorporating Technology." The examples in this text use TI-83/84 calculators, and the keystrokes will be identical on every available version of these calculators. Other models and brands of graphing calculators should function similarly. (The designation TI-83/84 refers to the TI-83, TI-84, TI-83+, TI-84+, TI-83+ Silver Edition and TI-84+ Silver Edition calculators.)

Consider the function $f(x) = x^3 - 2$. To graph $f(x)$, follow these steps:

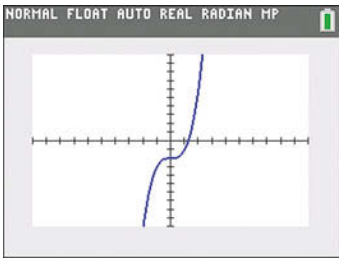
Step 1 Press [Y=]. We will define our function in the calculator as Y_1. Move the cursor up if necessary, so that it is placed directly after the expression "$\backslash Y_1 =$." Press [CLEAR] to ensure that no formulas are entered for Y_1.

Step 2 Enter $X\text{^}3 - 2$. The variable X may be entered using the [X,T,Θ,n] key. (See Fig. 10(a).)

Step 3 Press [GRAPH]. (See Fig. 10(b).)

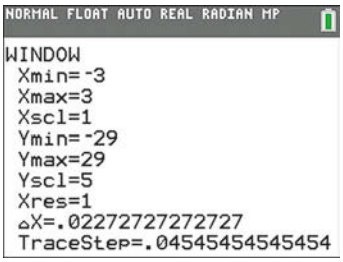


(a)

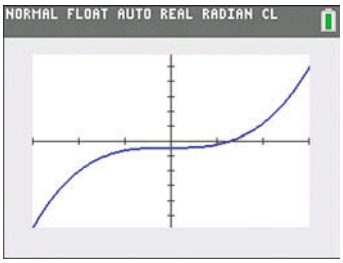


(b)

Figure 10



(a)



(b)

Figure 11

To change the parameters for the viewing window, follow these steps:

Step 1 Press [WINDOW].

Step 2 Change the parameters to their desired values.

One of the most important tasks in using a graphing calculator is to determine the viewing window that shows the features of interest. For this example, we will simply set the view to $[-3, 3]$ by $[-29, 29]$. We will also set the value of *Yscl* to 5. The parameter *Yscl* and its sibling, *Xscl*, set the distance between tick marks on their respective axes. To do this, set the window parameters on your calculator to match those in Fig. 11(a). The value of *Xres* sets the screen resolution, and we will leave it at its default setting.

Step 3 Press [GRAPH] to see the results. (See Fig. 11(b).)

Check Your Understanding 0.1

Solutions can be found following the section exercises.

1. Is the point $(3, 12)$ on the graph of the function $g(x) = x^2 + 5x - 10$?

2. Sketch the graph of the function $h(t) = t^2 - 2$.

EXERCISES 0.1

Draw the following intervals on the number line.

1. $[-1, 4]$ **2.** $(4, 3\pi)$ **3.** $(-\sqrt{5}, 4]$

4. $[1, \frac{3}{2}]$ **5.** $(-\infty, 3)$ **6.** $(4, \infty)$

Use intervals to describe the real numbers satisfying the inequalities in Exercises 7–12.

7. $2 \le x < 3$ **8.** $-1 < x < \frac{3}{2}$

9. $x < 0$ and $x \ge -1$ **10.** $x > -2$ and $x \le 5$

11. $x < 3$ **12.** $x \ge \sqrt{2}$

13. If $f(x) = x^2 - 3x$, find $f(0)$, $f(5)$, $f(3)$, and $f(-7)$.

14. If $f(x) = x^3 + x^2 - x - 1$, find $f(1)$, $f(-1)$, $f(\frac{1}{2})$, and $f(a)$.

15. If $f(x) = x^2 - 2x$, find $f(a + 1)$ and $f(a + 2)$.

16. If $g(t) = (t + 2)/(t - 5)$, find $g(-\frac{4}{5})$, $g(\frac{4}{5})$, and $g(k - 2)$.

17. If $f(x) = 3x + 2$ and $h \neq 0$, find $\dfrac{f(3+h) - f(3)}{h}$. Simplify your answer as much as possible.

18. If $f(x) = (x+1)^3$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$. Simplify your answer as much as possible.

19. **Temperature Scales** The boiling point of tungsten is approximately 5933 Kelvin.
(a) Find the boiling point of tungsten in degrees Celsius, given that the equation to convert x°C to Kelvin is $k(x) = x + 273$.
(b) Find the boiling point of tungsten in degrees Fahrenheit. (Tungsten, also known as wolfram, has the highest boiling point out of all known metals.)

20. **Mobile Sales** A mobile manufacturer finds that the number of mobiles sold in year x is given approximately by the function $f(x) = 225 + 5x + 2x^2$, where $x = 0$ corresponds to 2010.
(a) What does $f(0)$ represent?
(b) Find the number of mobiles sold in 2025.

In Exercises 21–24, describe the domain of the function.

21. $f(x) = \dfrac{8x}{(x-1)(x-2)}$

22. $f(t) = \dfrac{1}{\sqrt{t}}$

23. $g(x) = \dfrac{1}{\sqrt{3-x}}$

24. $h(x) = \dfrac{12}{(x+3)(x-5)}$

In Exercises 25–28, sketch the graph of the function.

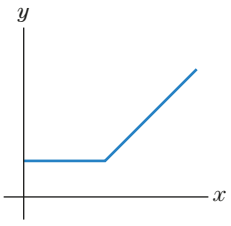
25. $f(x) = x^2 + 1$

26. $f(x) = 2x^2 - 1$

27. $f(x) = \sqrt{x+1}$

28. $\dfrac{1}{f(x) = x + 1}$

In Exercises 29–34, decide which curves are graphs of functions.

29.


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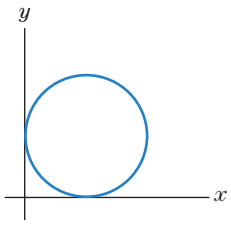

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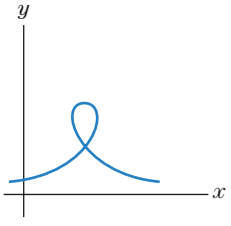

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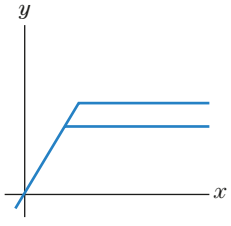

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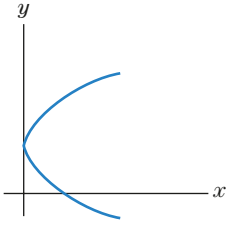

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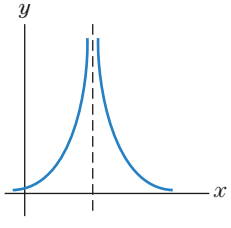

Exercises 35–42 relate to the function whose graph is sketched in Fig. 12.

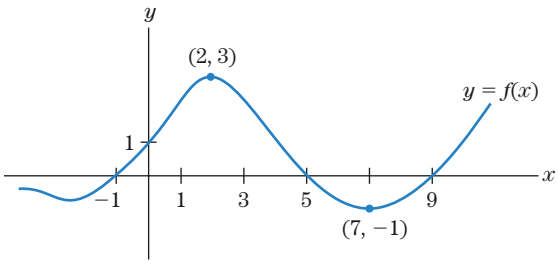

Figure 12

35. Find $f(0)$ and $f(7)$.

36. Find $f(2)$ and $f(-1)$.

37. Is $f(4)$ positive or negative?

38. Is $f(6)$ positive or negative?

39. What is the range of f?

40. For what values of x does $f(x) = 0$?

41. For what values of x is $f(x) \leq 0$?

42. For what values of x is $f(x) \geq 0$?

Exercises 43–46 relate to Fig. 13. When a drug is injected into a person's muscle tissue, the concentration y of the drug in the blood is a function of the time elapsed since the injection. The graph of a typical time–concentration function f is given in Fig. 13, where $t = 0$ corresponds to the time of the injection.

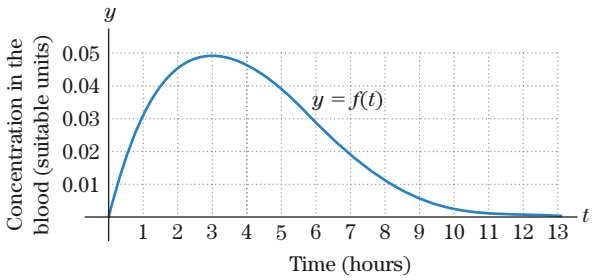

Figure 13 Drug time-concentration curve.

43. What is the concentration of the drug when $t = 1$ and $t = 5$?

44. What is the value of the time–concentration function f when $t = 6$?

45. What is the range of f?

46. At what time does $f(t)$ attain its largest value?

47. Is the point $(3, 12)$ on the graph of the function $f(x) = (x - \frac{1}{2})(x + 2)$?

48. Is the point $(-2, 12)$ on the graph of the function $f(x) = x(5 + x)(4 - x)$?

49. Is the point $(1, 1)$ on the graph of the function $g(x) = (3x - 1)/(x^2 + 1)$?

50. Is the point $(\frac{1}{5}, -5)$ on the graph of the function $h(x) = \dfrac{x^3 + 125}{x - 5}$?

51. Find the y-coordinate of the point $(a + 1, ?)$ if this point lies on the graph of the function $f(x) = x^3$.

52. Find the y-coordinate of the point $(2 + h, ?)$ if this point lies on the graph of the function $f(x) = (5/x) - x$.